LIESMARS

Course Title

Principles of Photogrammetry

Chapter 2:

FUNDAMENTAL GEOMETRY OF SINGLE AND OVERLAPPING IMAGES

GEOMETRY OF A CENTRAL PROJECTION

- **Definitions**
- **Central or perspective projection**
  - The projection of points from one plane onto a second (projection) plane, in which all projection lines pass through a single point
- **Orthogonal projection**
  - One in which all projection lines are at right angles to the projection plane
  - The geometry of a central projection is significantly different from that of a map

CENTRAL PROJECTION

- Assume that a set of rays pass through a single point.
- If this set of rays is cut by two planes, so that the intersections of the rays on the two planes are A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 respectively, then we have two sets of points which form corresponding figures in the two planes.
- One plane can be described as a central projection of the other.

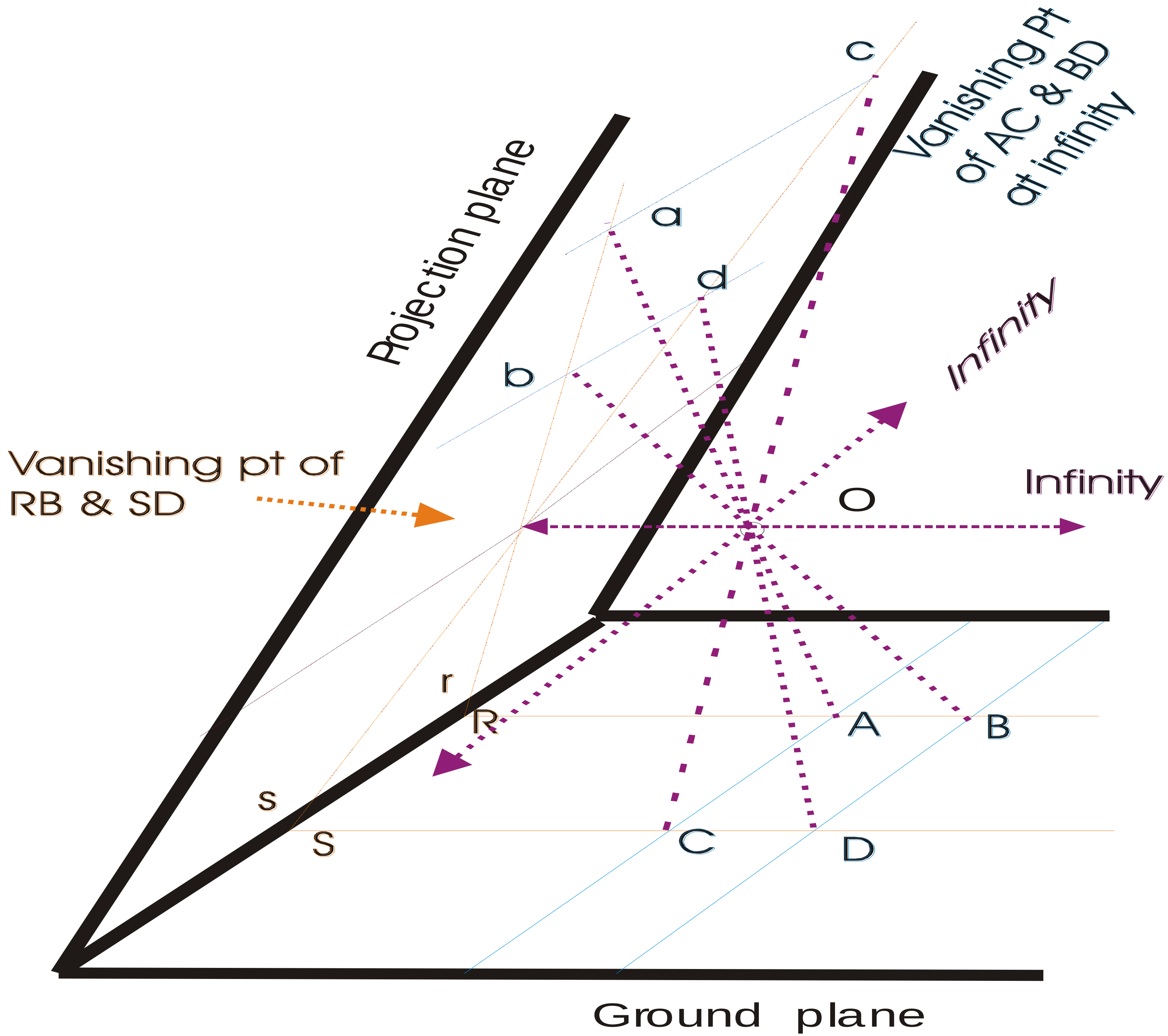


Figure 2.1 Perspective projection of horizontal lines

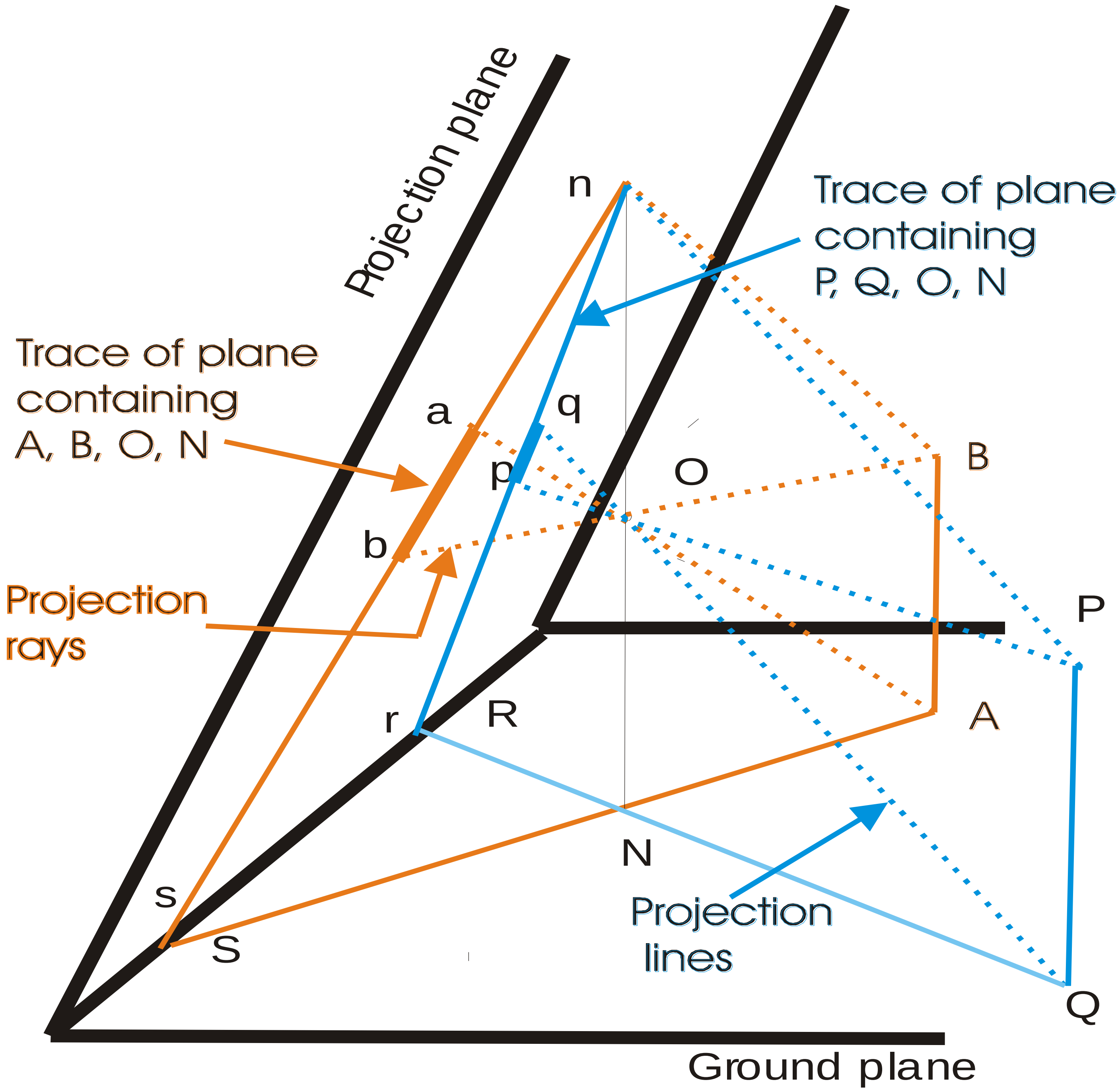


Figure 2.2 Perspective projection of vertical lines by constructing vertical planes containing the given line, the perspective centre, and the nadir point N

Properties of a Central Projection

- Straight lines on one plane are straight lines in the projection.
  - Hence, straight lines on a horizontal ground plane are straight lines on a photograph.
  - Note that if elevation differences occur on the ground then this rule does not apply, as described later.
- Parallel straight lines on one plane (eg the horizontal ground plane) will be projected as straight lines that intersect at the vanishing point.
- The vanishing point of horizontal lines on one plane (eg ground plane) will lie on the horizon on the projection.
- Vanishing Point of vertical lines is the Nadir Point in the projection

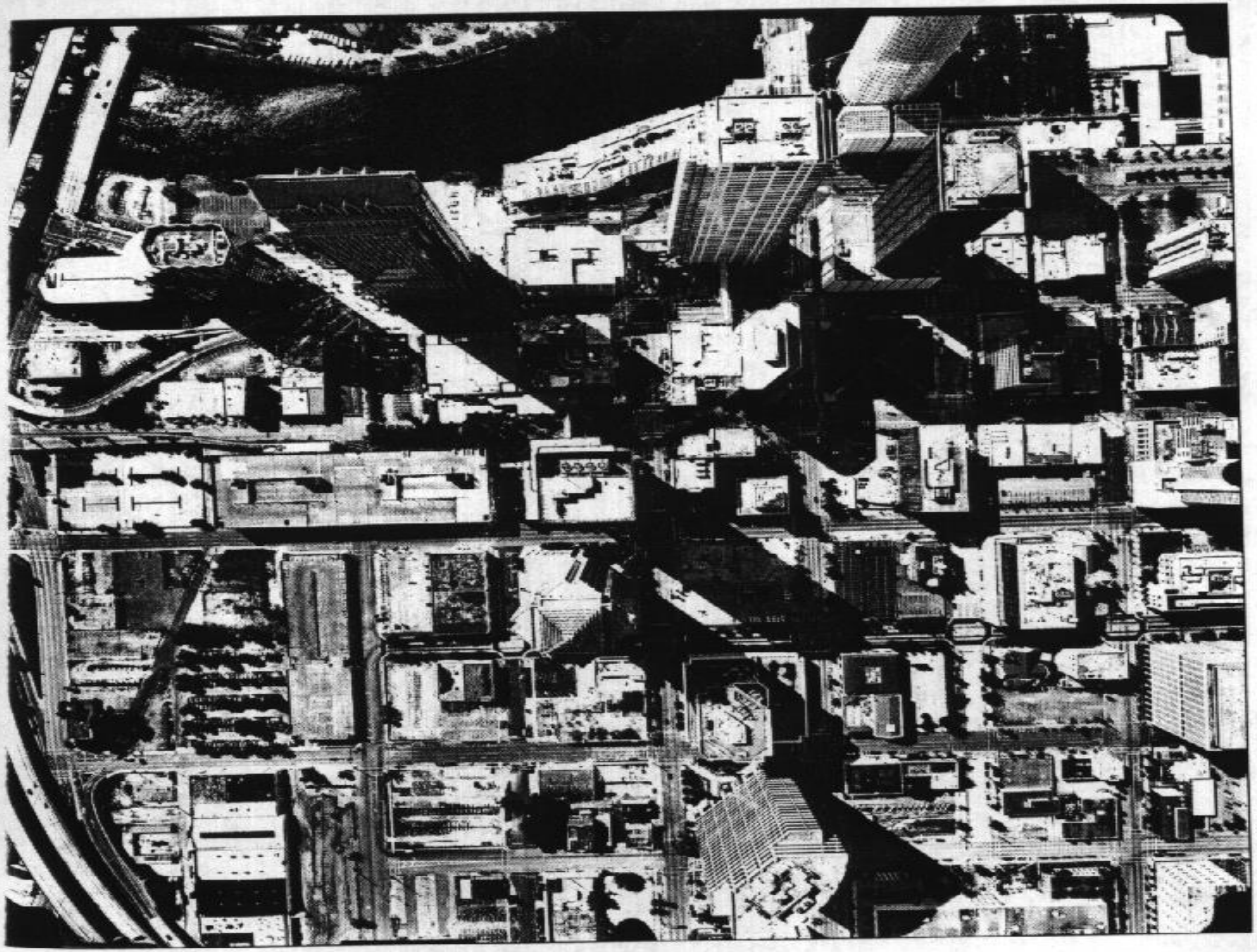
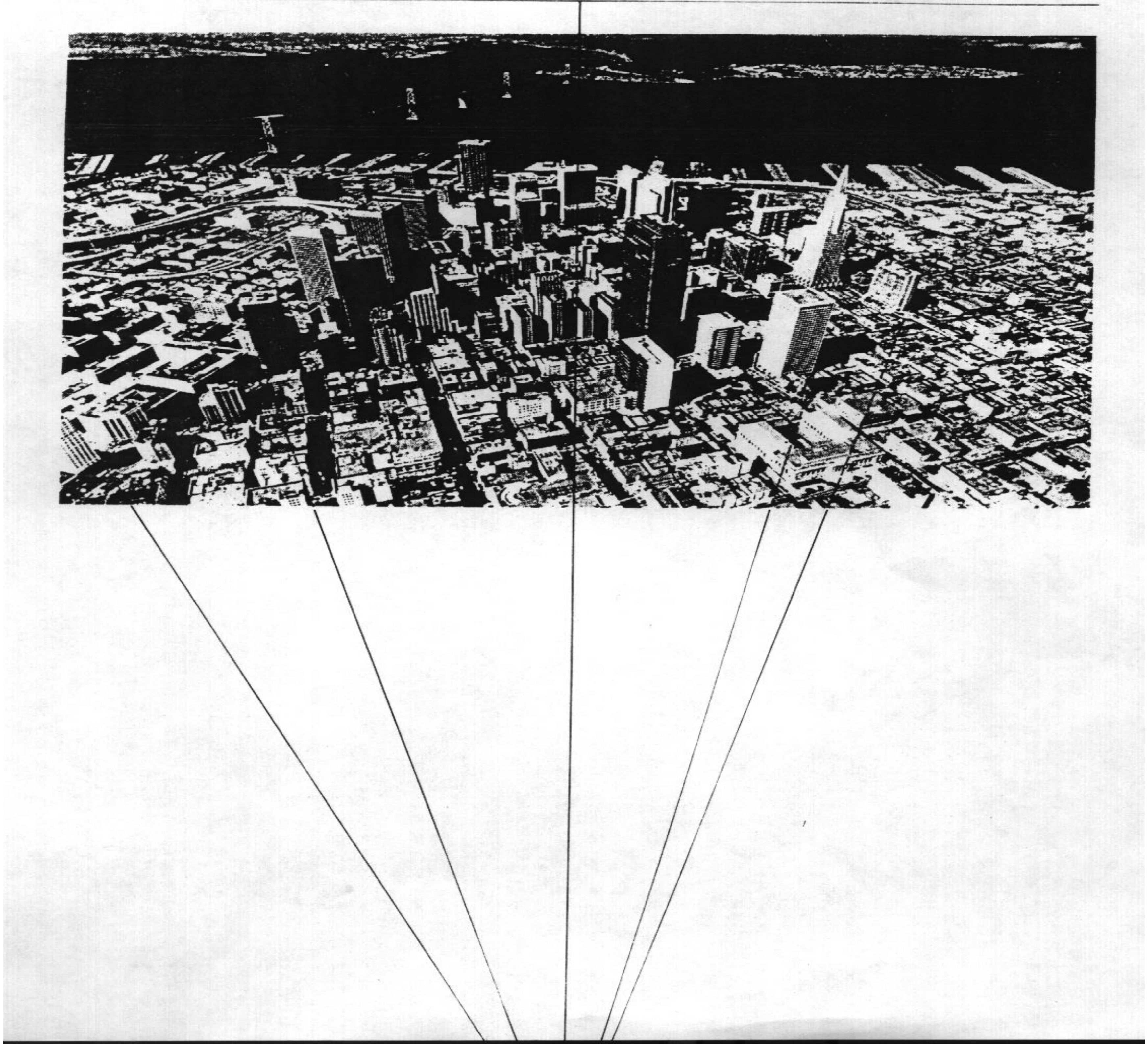


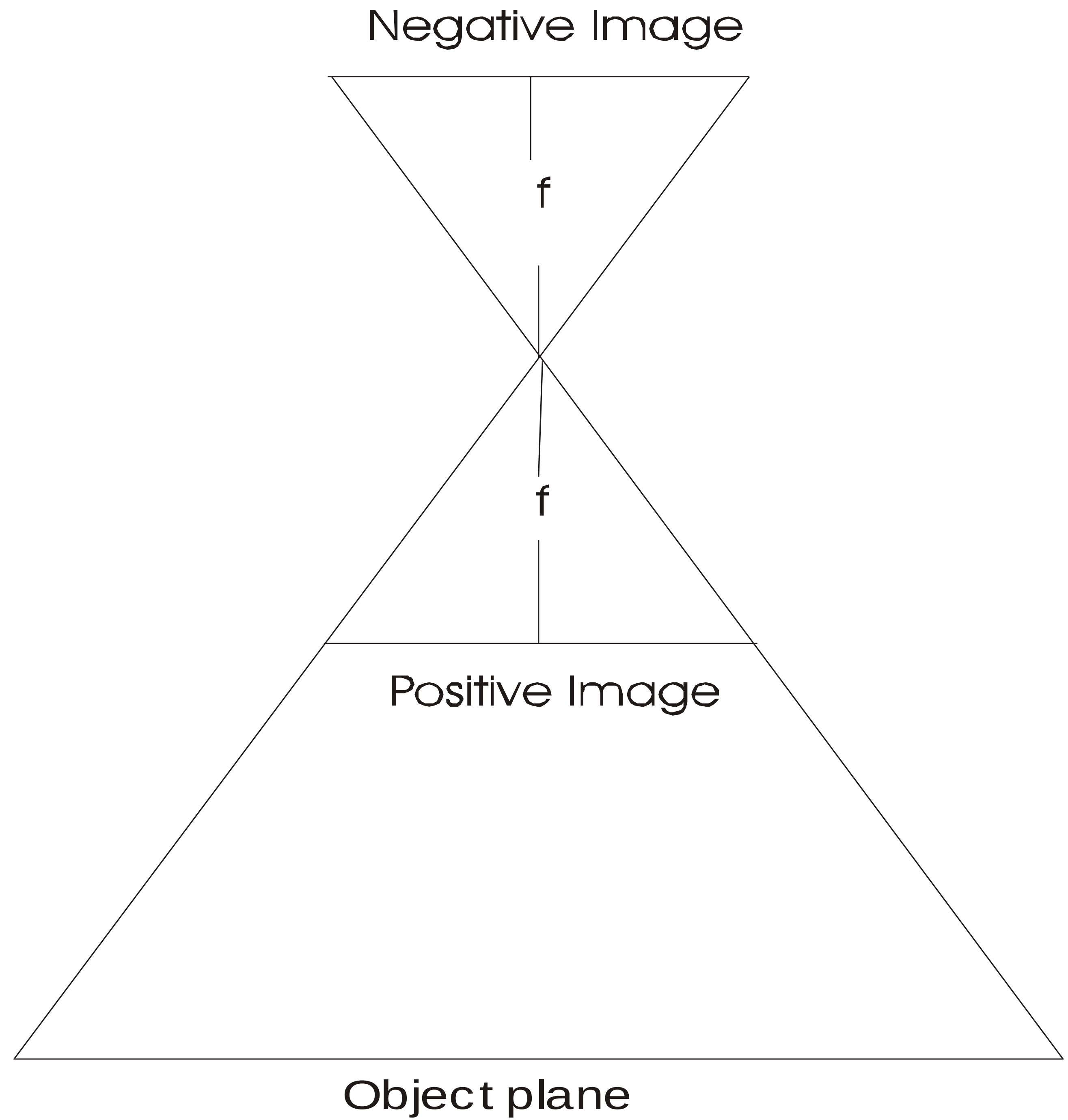
FIGURE 6-7
Vertical photograph of Tampa, Florida, illustrating relief displacements. (*Courtesy US Imaging, Inc.*)




07_RT8_A, d=3897.7, an=43.0
05_RT8_A, d=3857.9, an=43.9
09_RT8_A, d=3716.0, an=44.2
04_RT8_A, d=4031.4, an=39.9

Significant Points on the Photograph

- The Principal Point is the point at the foot of the perpendicular from the perspective centre to the image plane.
- The Nadir Point is the point at the intersection of the image plane and the plumb line.
- The Isocentre is the intersection of the bisector of the angle at the perspective centre formed by the lines to the principal point and the nadir point.


Negative Image
f
f
Positive Image
Object plane

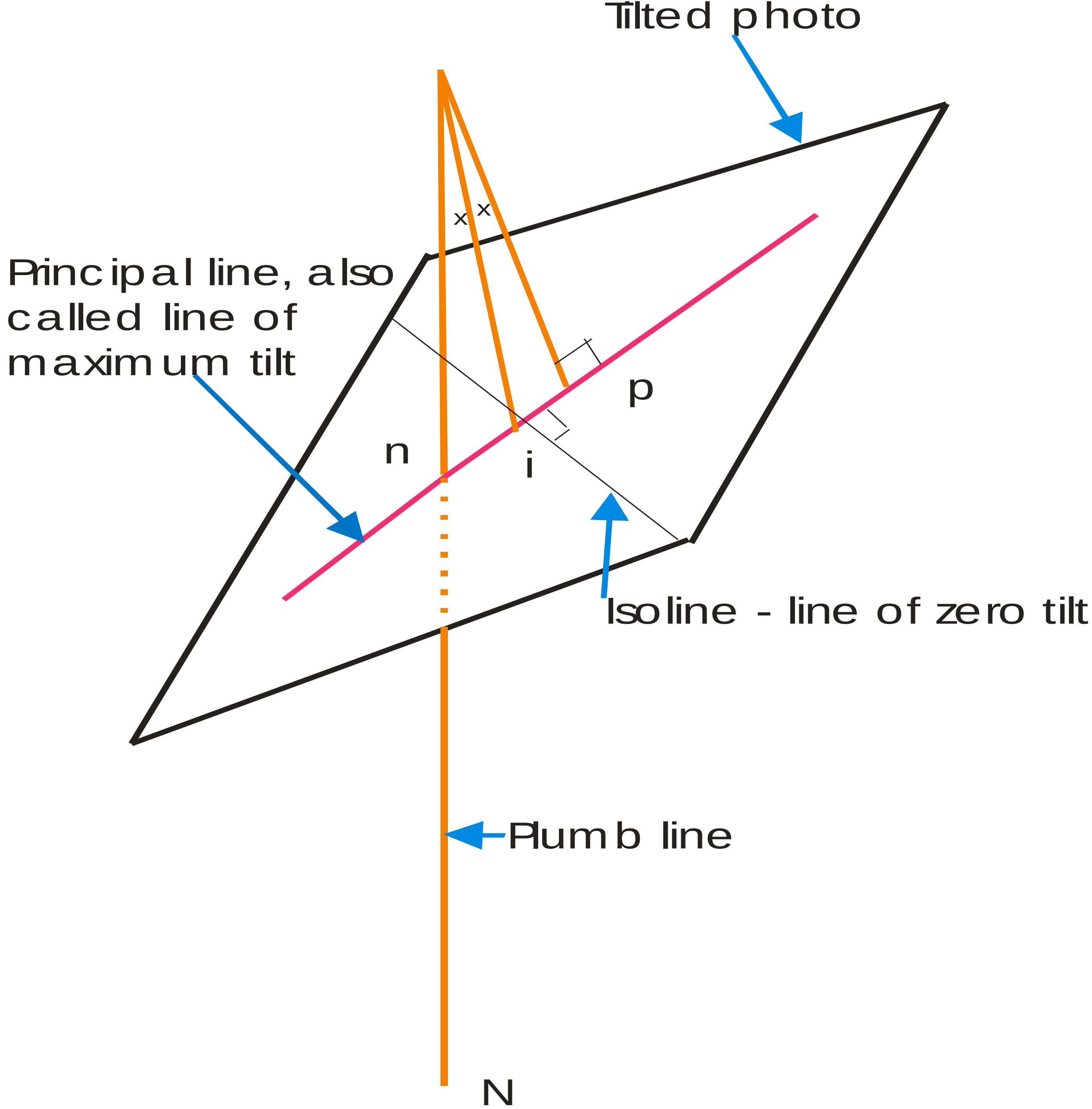


Figure 2.3 Tilted photo showing principal point p, isocentre I, and nadir point n.

Scale of an exactly vertical aerial photograph

$$\text{Scale} = \frac{f}{Z - h}$$

f is the principal distance of the photograph

Z is the flying height above datum

h is the local elevation above datum of a point on the image

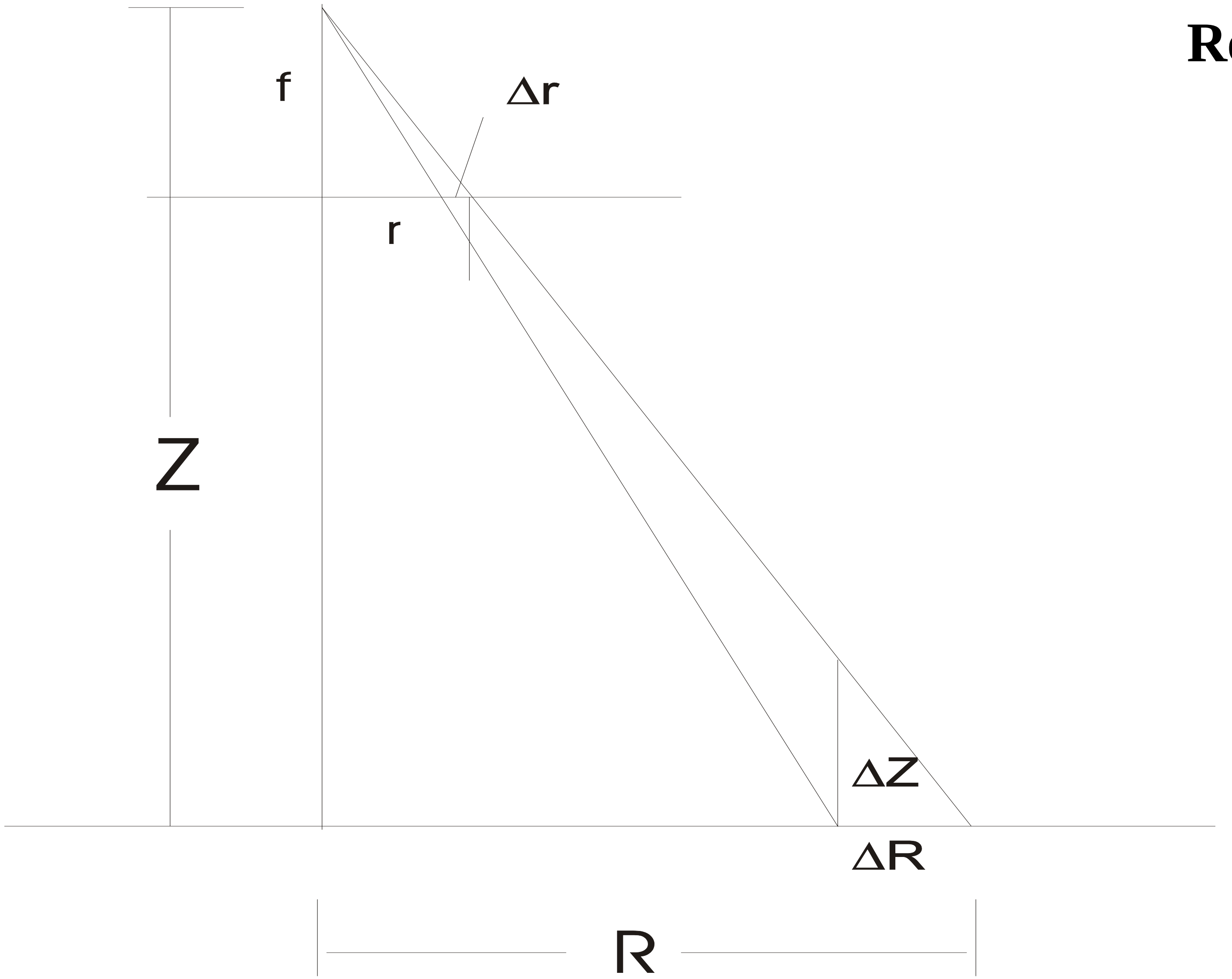
The scale will vary over the photograph according to the elevation of the points on the ground.

Scale is always expressed as 1:S, where S is the scale number

Displacements on an Aerial Photograph

There are 2 major sources of geometric **displacements** on an aerial photograph, due to

- relief differences in the terrain
- tilts of the photograph.



Relief displacement

$$\Delta r = \frac{\Delta Z . r}{Z}$$

The magnitude of these distortion can be determined by substituting appropriate values of the parameters

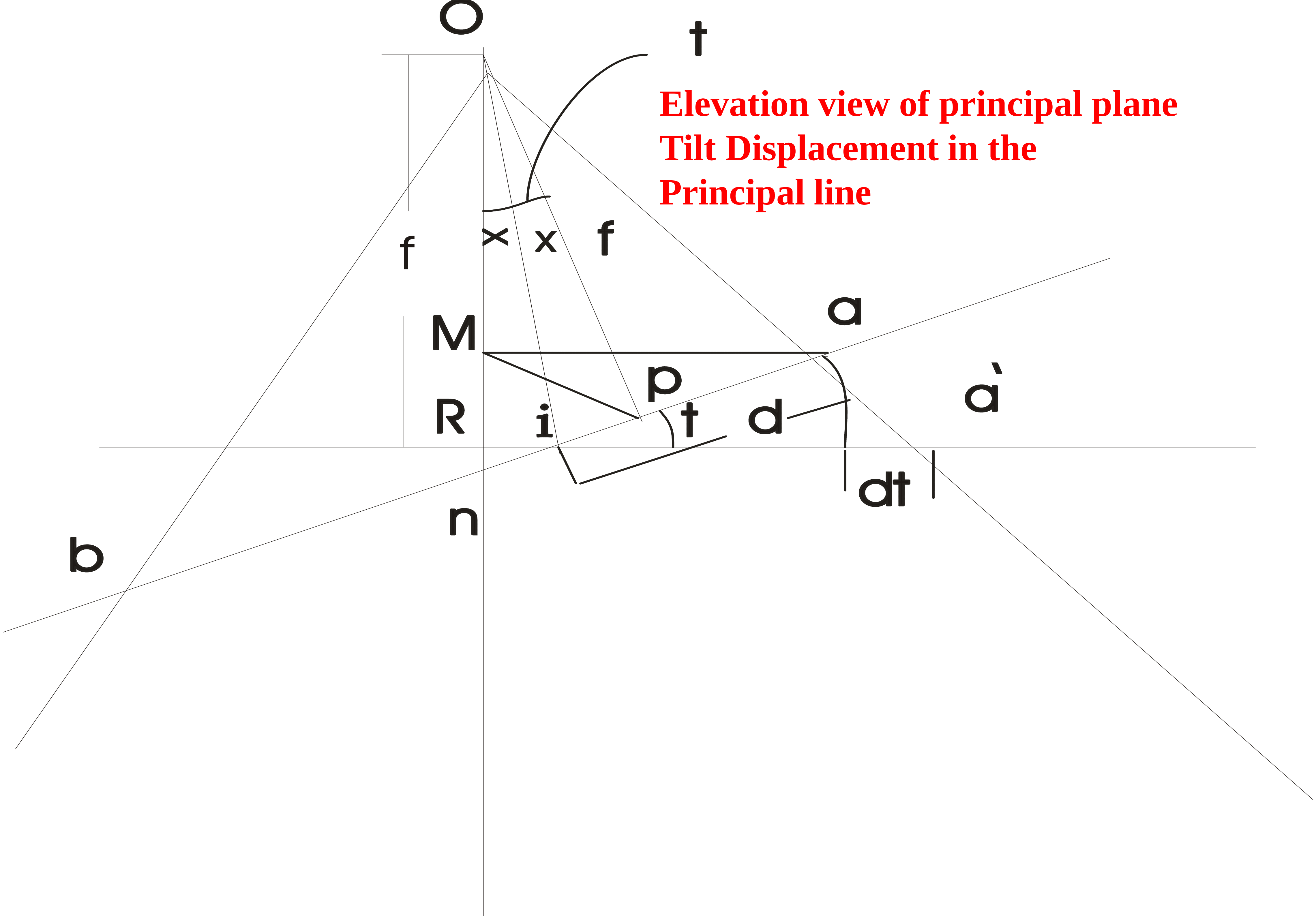
$\Delta Z = 100$ m

$r = 100$ mm

$Z = 2000$ m

Hence

$\Delta r = 5$ mm


O
t
Elevation view of principal plane
Tilt Displacement in the
Principal line
x
x
f
f
a
M
p
a`
R
i
t
d
dt
n
b

Tilt Displacement at ‘a’ in the principal line on high side of photo

$$dt = \frac{d^2 . \sin t}{f - d . \sin t}$$

or

$$dt = \frac{d^2}{\frac{f}{\sin t} - d}$$

$$dt = \frac{d^2 . \sin t}{f}$$

Direction of tilt displacement

- **The tilt displacement is:**

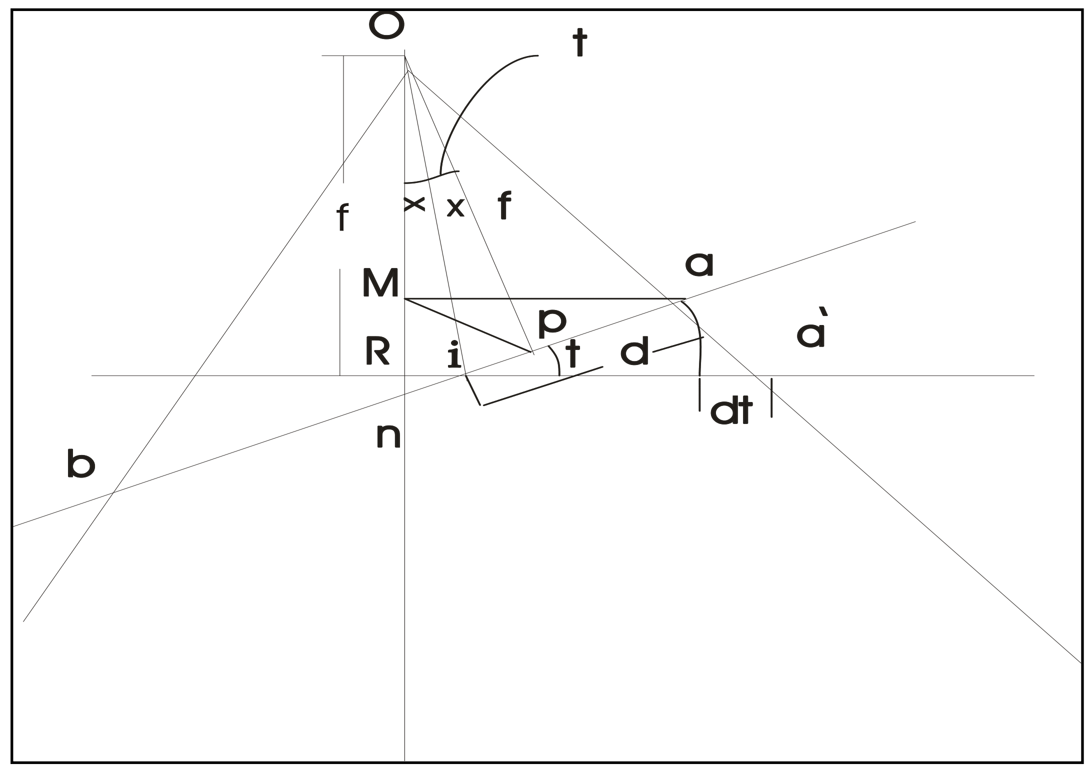
  - towards the isocentre, for point ‘a’ on the 'high side' of the photograph.
  - away from the isocentre for point ‘b’ on the 'low side' of the image.

Tilt displacement in the principal line on low side of photograph

$$dt = \frac{d^2 . \sin t}{f + d . \sin t}$$

or

$$dt = \frac{d^2}{\dfrac{f}{\sin t} + d}$$



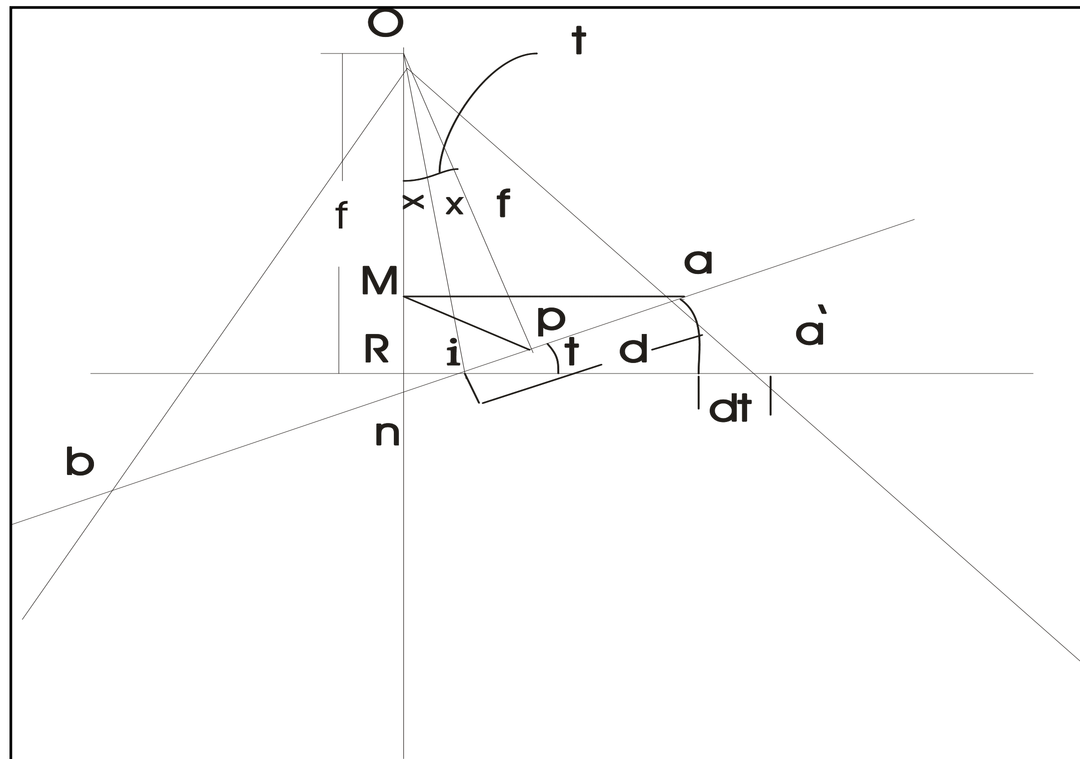
The scale in the principal line of the tilted photograph

For point ‘a’ on the 'high side' of the photograph

$$\text{Scale}_a = \frac{f - d.\sin t}{Z_a}$$

For point ‘b’ on the ‘low side' of the photograph

$$\text{Scale}_b = \frac{f + d.\sin t}{Z_b}$$



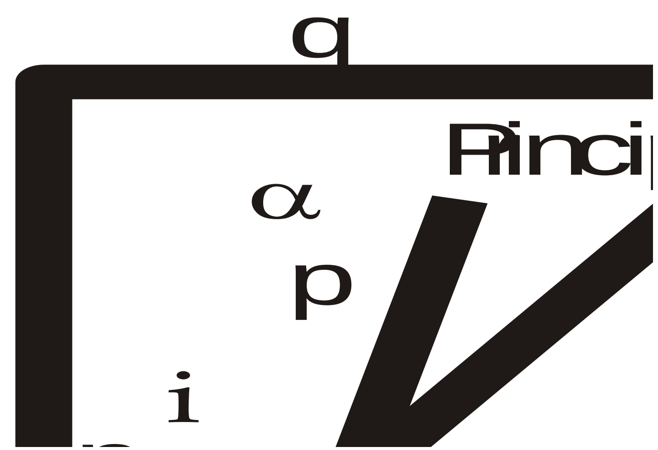
Tilt displacements for points not on the Principal Line

The tilt displacement of any point q, which is not on the principal line, is derived by determining the tilt of the line from q to the isocentre.

This is derived from the angle α at the isocentre between the principal line and the line to point q.

The approximate tilt t' of this line is derived from

$$\sin t' = \sin t . \cos \alpha$$



An example of tilt displacement

- Photo tilt is 2°, point 'a', is 70 mm from the isocentre along the principal line. The principal distance is 150 mm. Find the tilt displacement at a.
- Find the tilt displacement for a point 'q' which is 70 mm from the isocentre on a line at an angle of 25° to the principal line.
- Answer:
- Tilt displacement at:

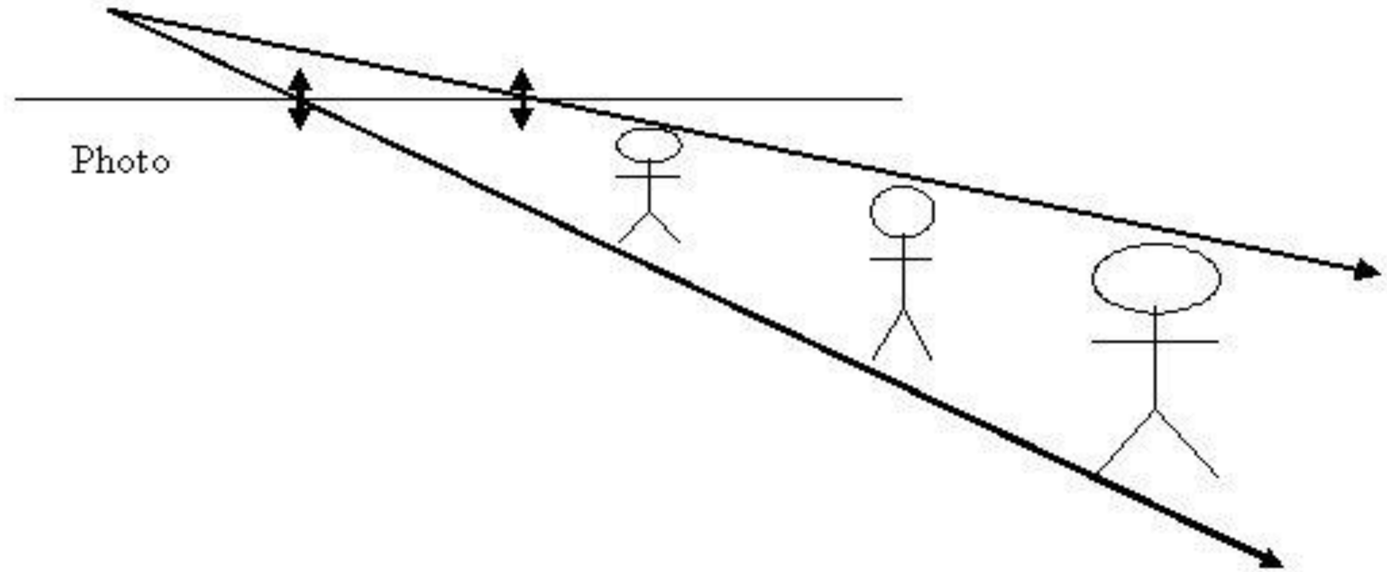
a = 1.16 mm

q = 1.04 mm

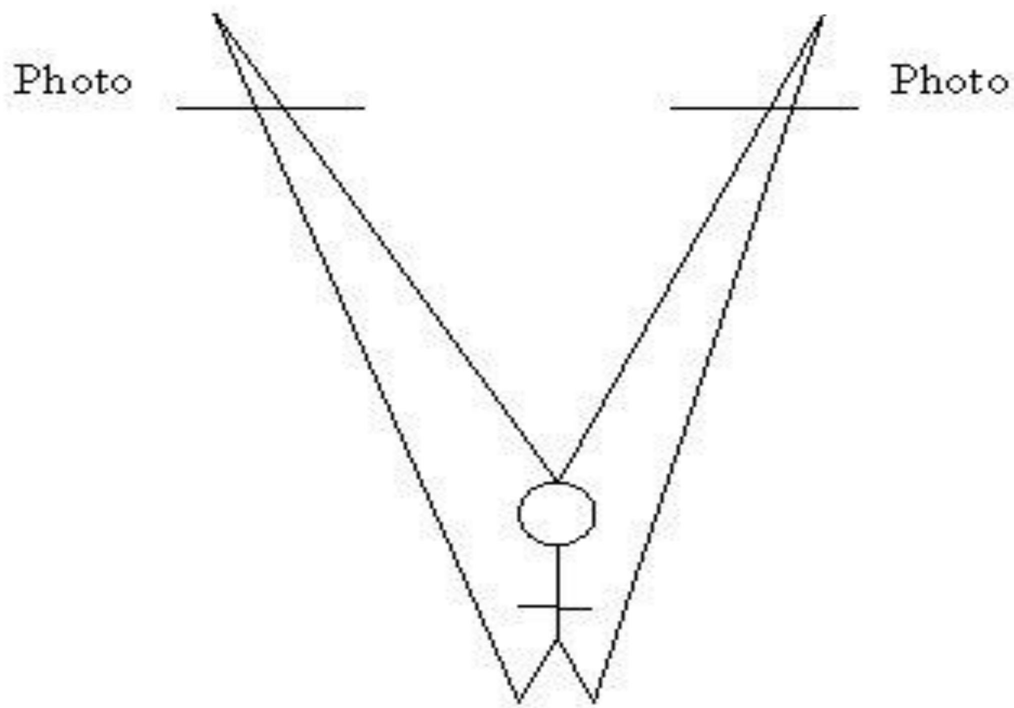
Principles of Measurements from Overlapping Photographs

- Accurate measurements by photogrammetry are derived from 2 or more overlapping photographs or digital images

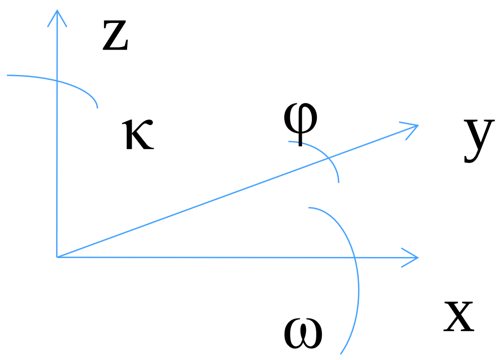
Measurement of objects
on a single image –
no scale available

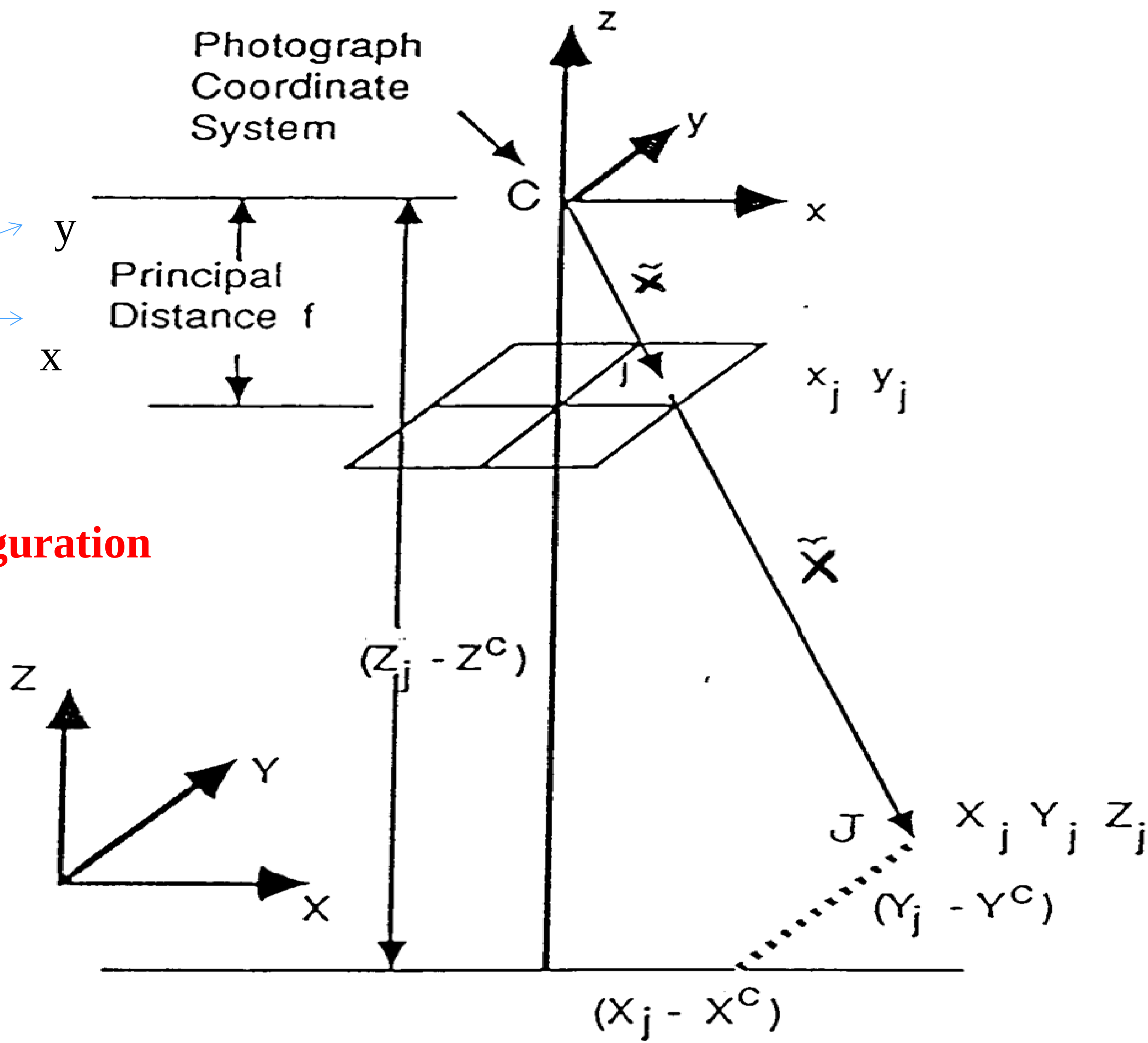


Comparison of measurements on 1 and 2 photos



Measurement of objects
on 2 photos

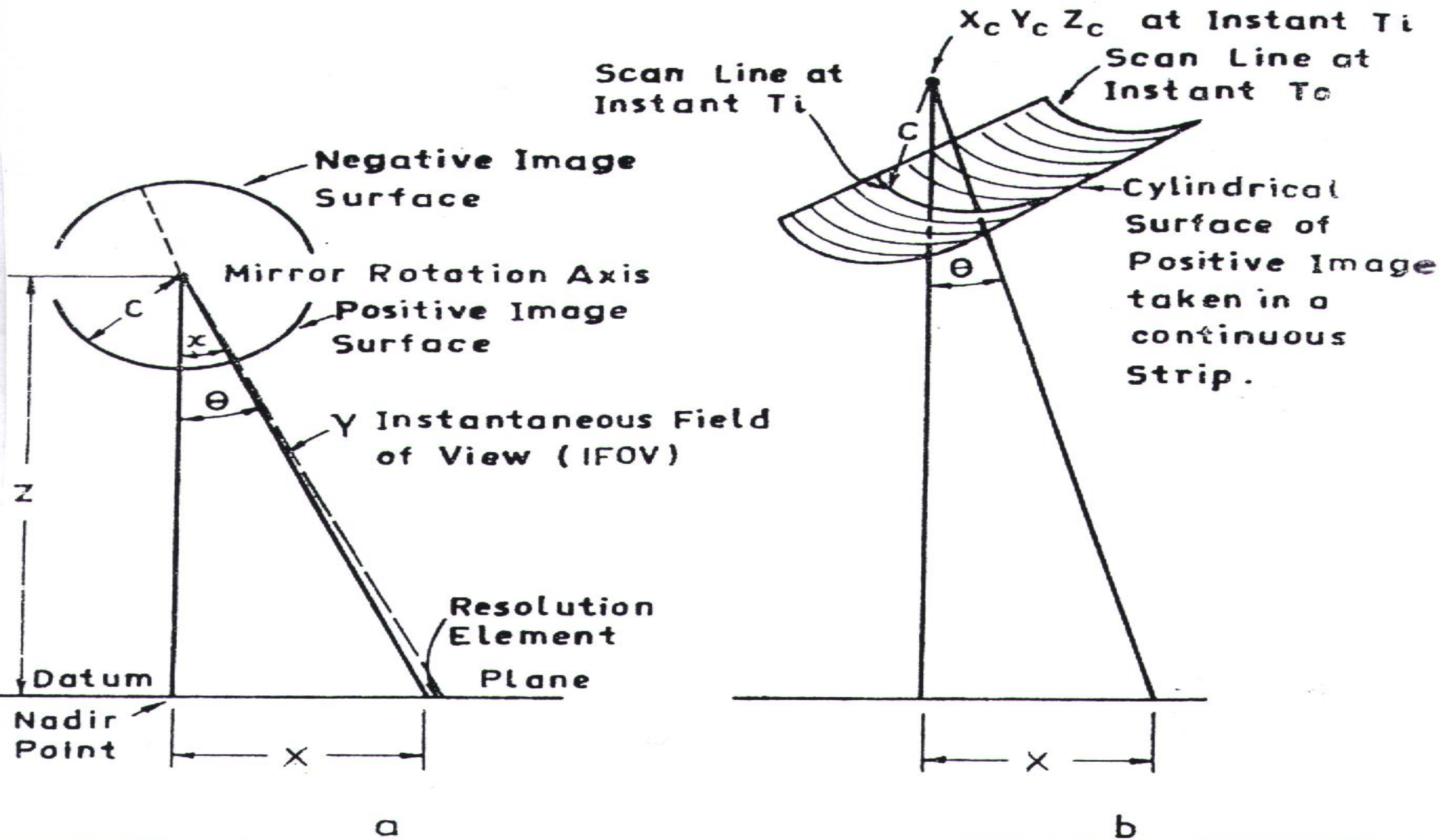




Fundamental configuration for single photo

Geometry of Whisk-broom

- y-axis is along the flight direction
- x-axis is at right angles
- the image is a mosaic of successive scan lines
- y-coordinate is dependent only on the elapsed time along the flight
- x-coordinate is dependent on the scan angle from the vertical



Negative Image Surface
Mirror Rotation Axis
C
x
Positive Image Surface
θ
Y Instantaneous Field of View (IFOV)
Z
Resolution Element
Datum
Plane
Nadir Point
X
a
$X_C\,Y_C\,Z_C$ at Instant Ti
Scan Line at Instant Ti
Scan Line at Instant To
C
Cylindrical Surface of Positive Image taken in a continuous Strip.
θ
X
b

For scan line i, taken at time T_i and assuming no tilts of the aircraft

$$Y_i = Y_0 + V(T_i - T_0)$$

where

Y_0 is the coordinate on the ground at the commencement of the flight line at time T_0

V is the aircraft speed assuming that it is constant

The image x-coordinate is

$x = c.\theta$

where c is a constant of the scanner

θ is the instantaneous angle of rotation of the mirror

- Therefore, for a flying height of Z, the X ground coordinate is given by

$$X = Z.\tan\theta = Z.\tan(x/c)$$

- No simple relationship between the image and ground coordinates in a mirror whisk-broom scanner,
- Large 'panoramic' distortions occur in the direction of scan
- Must be corrected by resampling

Figure 3a
Original Sample of Infrared Imagery Rectified.

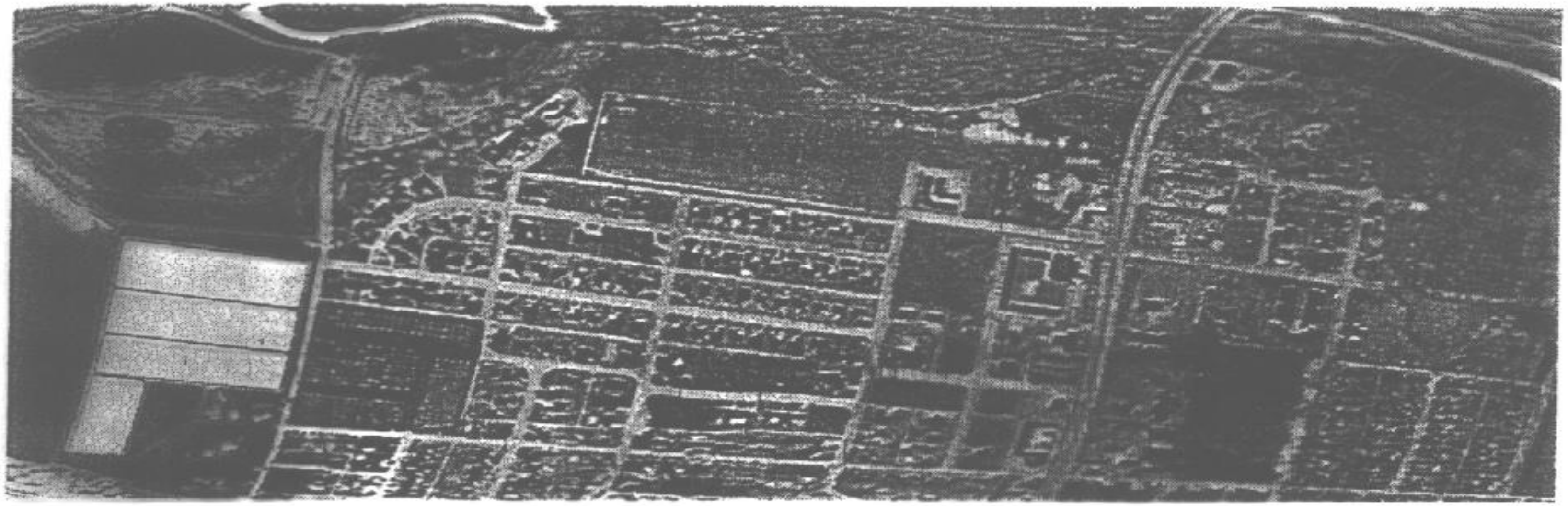
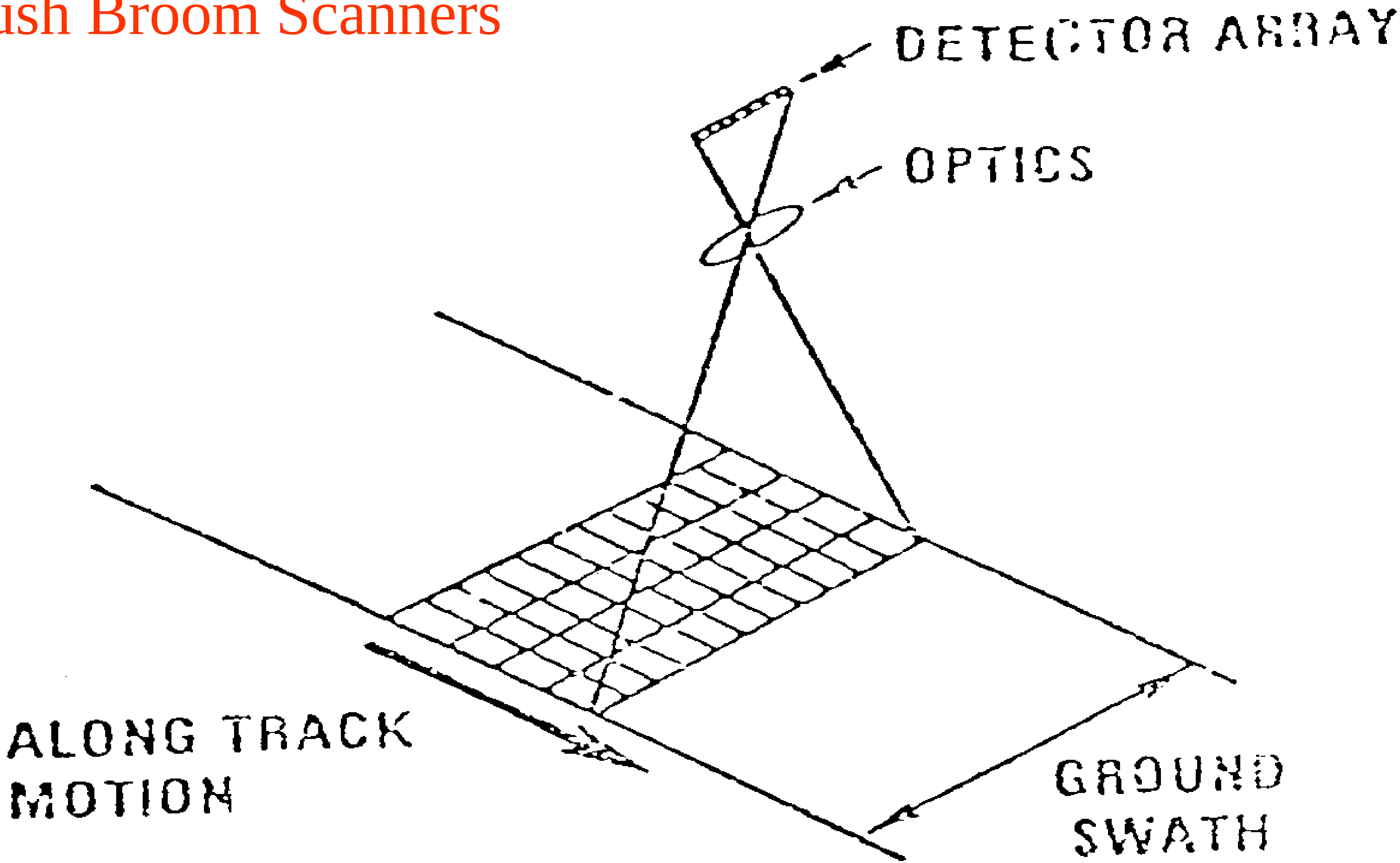


Figure 3b
Rectified Infrared Imagery Sample (see Figure 3a).

(by Courtesy of S.E. Masry & J. G. Gibbons).

Push Broom Scanners



Pushbroom scanners

- The geometry along each scan-line acquired by CCD array can be treated in a similar manner to photos.
- The coordinates along the flight direction must be treated in a similar manner to mirror scanners.
- The scale of the image along a scan line can be treated in a similar manner to aerial photography
- Push-broom images will be affected by the elevation of the terrain and the tilts of the platform
  - Elevations: similar to relief displacements
  - Tilts: vary as the platform moves over the terrain
- An analytical approach to the computation of 3D coordinates from pushbroom sensor images will be given in Chapter 3

Stereovision

DIRECT STEREOVISION

- Characteristic of normal binocular vision, created by an observer having two eyes that are separated by the 'eye-base'.
- An observer perceives 3 dimensions by the following cues:
  - Retinal disparity - difference between the images received by each of the eyes, because of their separation.
  - Geometric perspective
  - Relative sizes of objects
  - Objects being obscured by others
  - The extent of variation in accommodation and convergence of the eyes

DIRECT STEREOVISION

- The cue used will depend on the distance of the viewer from the object and other factors
- If an observer has an eye defect, so that the two images cannot be fused, then 3D vision will not be possible.
- Stereo vision was an important skill in the past for manual operations in photogrammetry
- It is less important for the processing of digital images, which are normally processed to digital orthophotos.

INDIRECT STEREOVISION

- Photogrammetry uses stereo vision in an indirect way. Images recorded by a different means, eg from 2 separate camera positions, are viewed by the eyes.
- Each of the eyes must see only one image, in the same way as the eyes see separate images by direct stereo vision.
- The impression of relief of the object is usually substantially exaggerated compared to the object seen by the observer's own eyes

STEREOSCOPIC EXAGGERATION

Stereoscopic exaggeration is primarily caused by:

- The 'base' between the photographs being much larger than the eye-base
- Optical magnification that is applied to the observation.
- The approximate stereoscopic exaggeration, n, can be derived from the ratio of

$$n = \frac{base_{photo}}{eyebase} * \ magnification$$

The actual exaggeration is unknown since it is a function of the observer's own perception of the object.

Indirect stereovision is obtained by viewing pairs of photos under the following conditions:

1. Each eye must receive only one image

2. Rays to corresponding points must intersect, that is, parallax between the images must be eliminated

Aids used for obtaining stereovision

- Stereoscopes
- Anaglyph system using images coloured by complementary colours
- Polarised images
- Lenticular displays

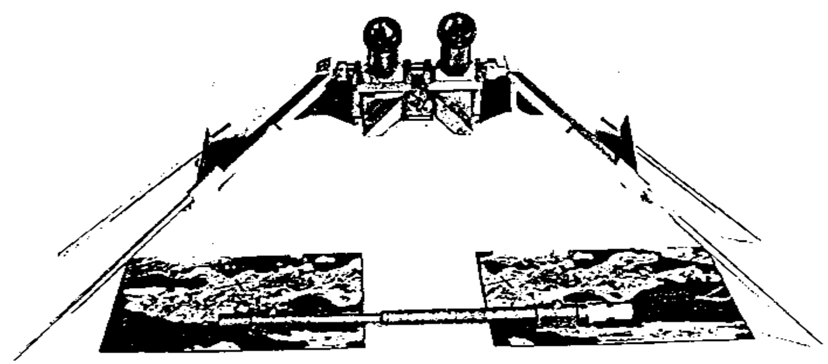


Figure Mirror stereoscope. Courtesy of Wild Heerbrugg Instruments, Inc.

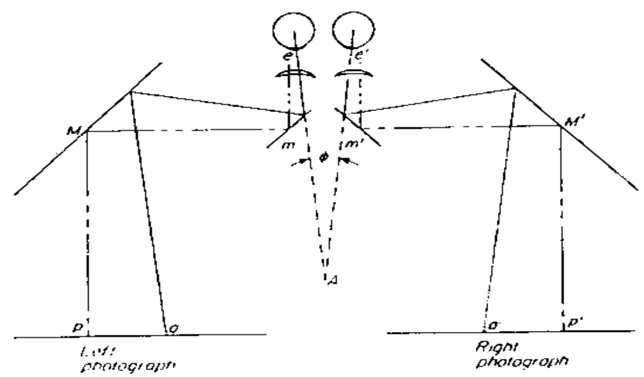


Figure Principle of mirror stereoscope.

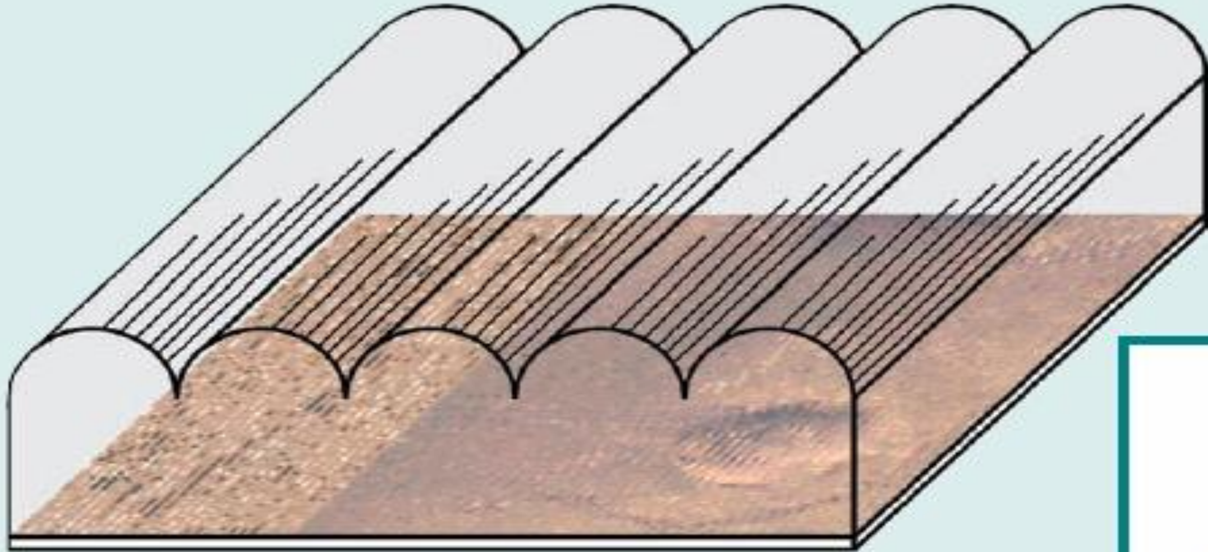


Figure 1. Structure of lenticular displays.

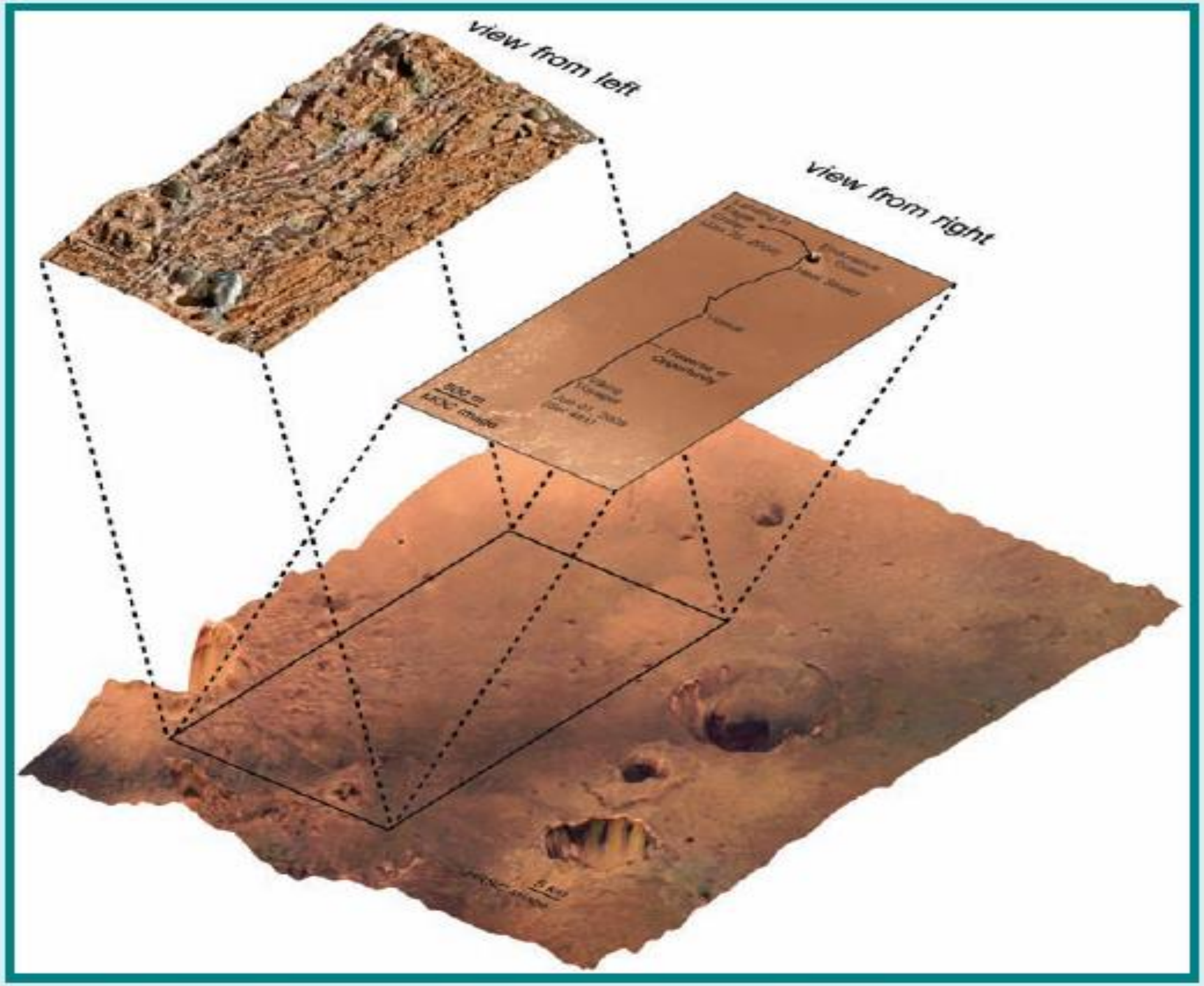
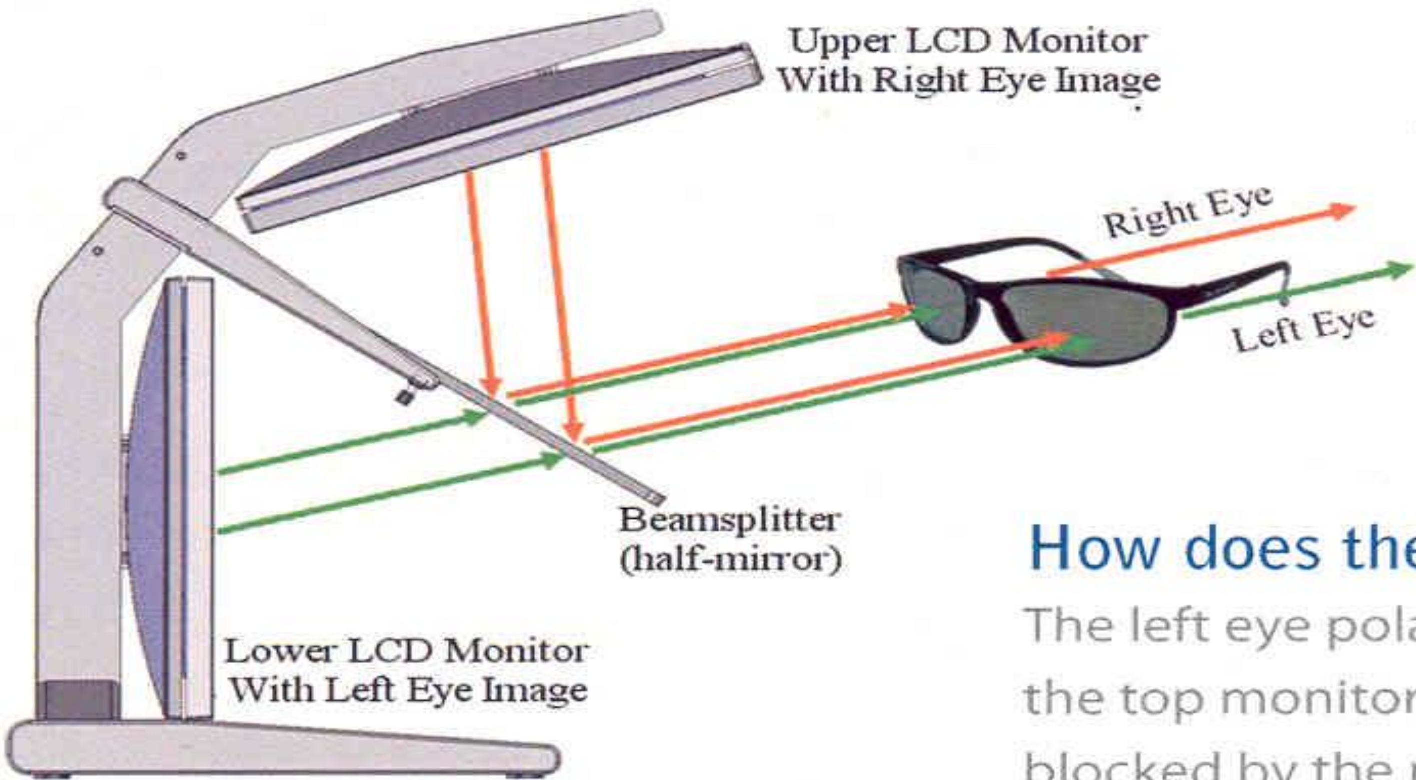


Figure 2. Principle of perception of true-3D/flip lenticular display.



How does the StereoMirror work?

The left eye polarized lens of the glasses blocks light from the top monitor while light from the lower monitor is blocked by the right eye lens.

Conditions of photography for comfortable viewing

- Photos must be of approximately the same scale within 10%, preferably 5%
- Camera axes must be approximately parallel
- Angles between camera axes must be less than 90°

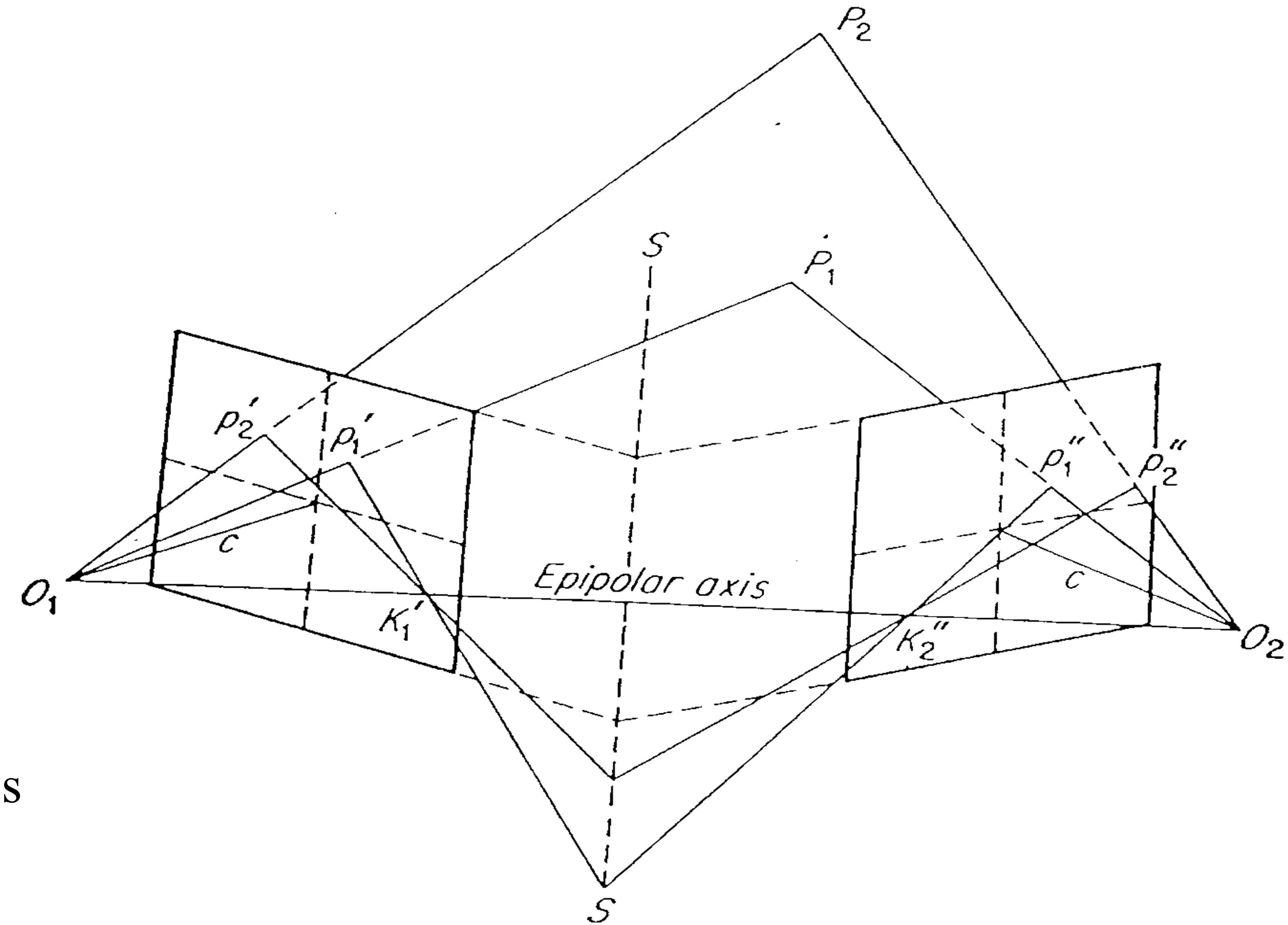
Epipolar Principle

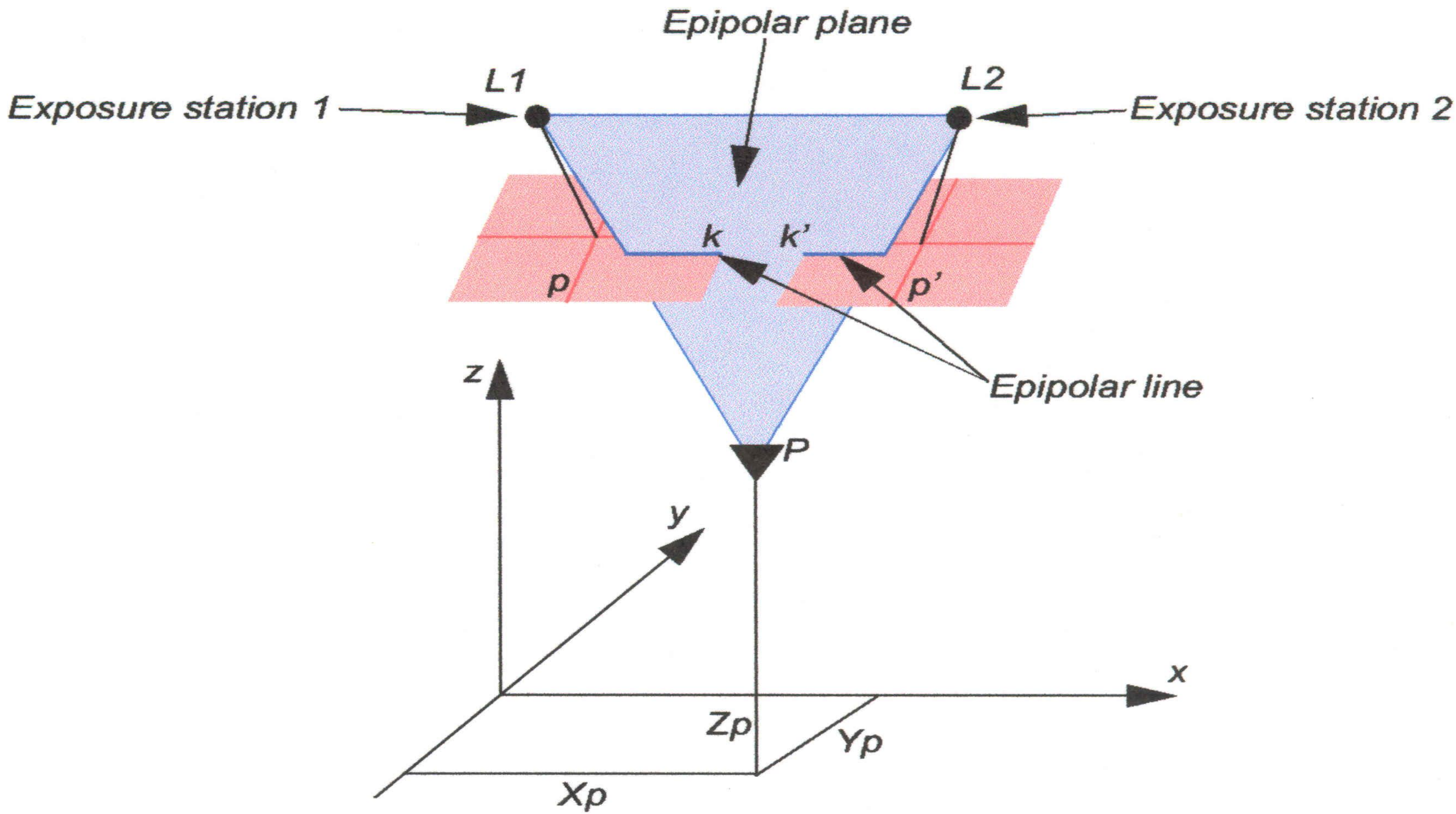
The **epipolar axis** is defined as the line between the perspective centres O_1 and O_2 (also called the photo base or airbase).

The **epipoles** $K'_1$are K''_2 intersections of epipolar axis and positive photo planes

Epipolar plane contains epipolar axis and an object point

Epipolar rays are intersection of epipolar planes and positive photos $p_1K'_1$ and $p''_1K''_2$




Epipolar plane
L1
L2
Exposure station 1
Exposure station 2
k
k'
p
p'
Epipolar line
z
P
y
x
Zp
Yp
Xp

SETTING UP A STEREOSCOPE

A stereoscope is the simplest instrument for viewing a pair of overlapping photographs.

It must be set up correctly or the observations will be uncomfortable

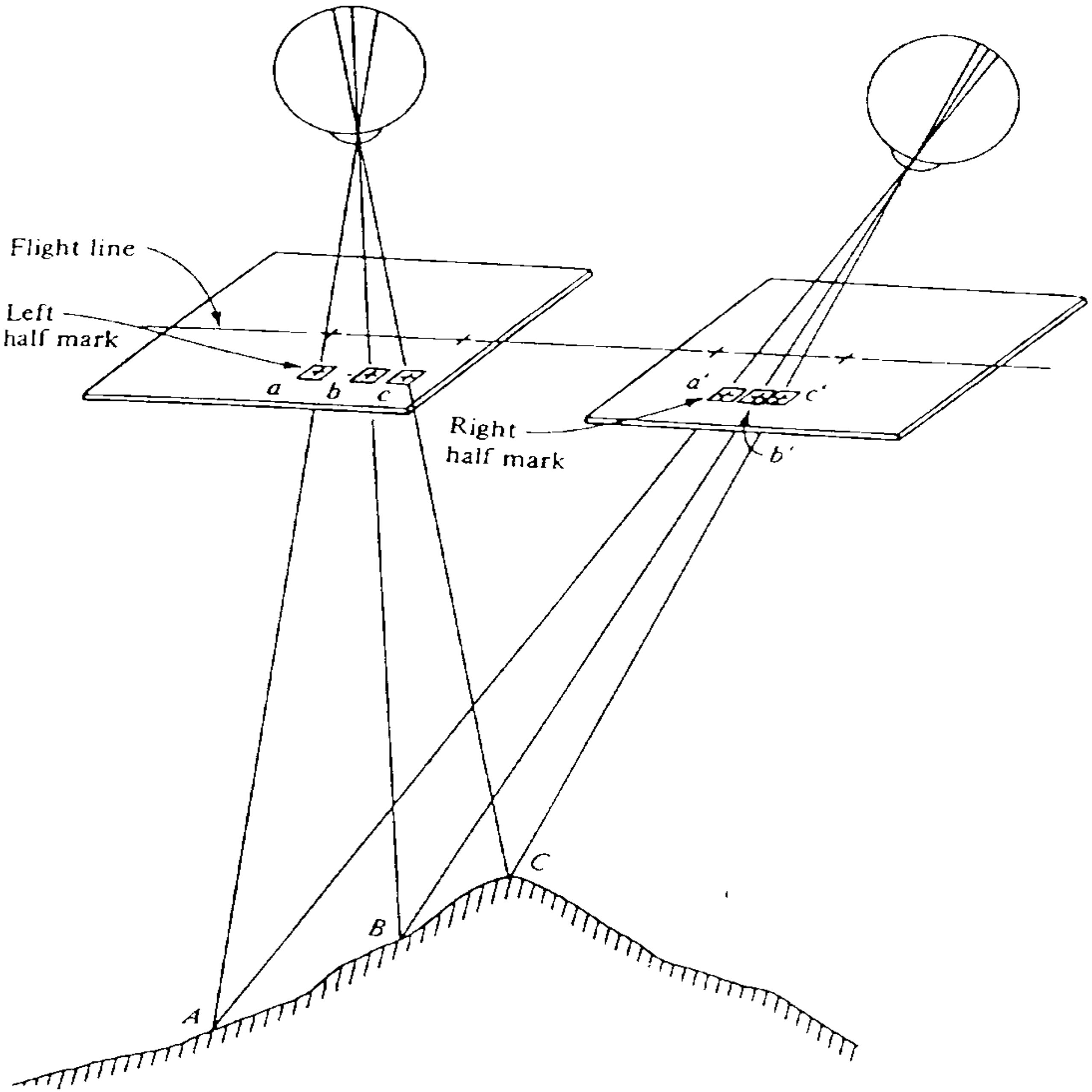
SETTING UP A STEREOSCOPE FOR HARDCOPY IMAGES

Procedure

1. Mark on the photographs, the principal points and conjugate principal points
2. Place the two photos on a board and align the principal points and the conjugate principal points
3. Place the stereoscope over the photos, and with the ruler placed over the principal and conjugate principal points, rotate the stereoscope until the images of the ruler in the two oculars of the stereoscope coincide
4. The correct separation of the photographs is established by sliding the unfastened photo, maintaining the alignment, until the images coincide. Fasten the second photo to the board
5. Observe the photos by adjusting the rotation of the stereoscope with respect to the photos

The floating mark

A floating mark is used to make measurements on a pair of photographs



Points A, B, and C on the terrain are photographed as a, b, c on the left hand photo and a', b' and c' on the right photo

If an observer views the photos by indirect stereo observation, the terrain will be seen in 3D, with the correct relative positions of A, B, C

If two small marks are placed into the observation system, one coinciding with **a** on the left photo and the other with **a'** on the right photo, the two marks are viewed stereoscopically, the combined mark will appear to coincide in 3D with the actual point A on the terrain

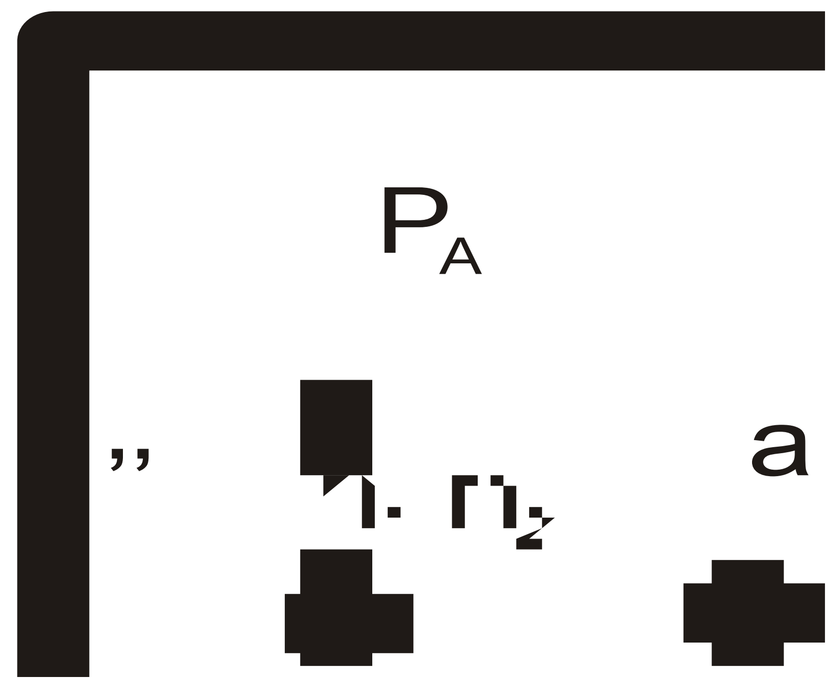
These points are referred to as 'floating marks', since they are seen in 3D and by varying the separation of the two points the floating marks can be seen as either as floating above the ground or sitting below the ground

Floating marks are the measuring marks in photogrammetry for hardcopy and digital images

PARALLAX

Stereoscopic parallax is caused by a change in position of a point on two consecutive photos due to a change in position at which the photo is taken

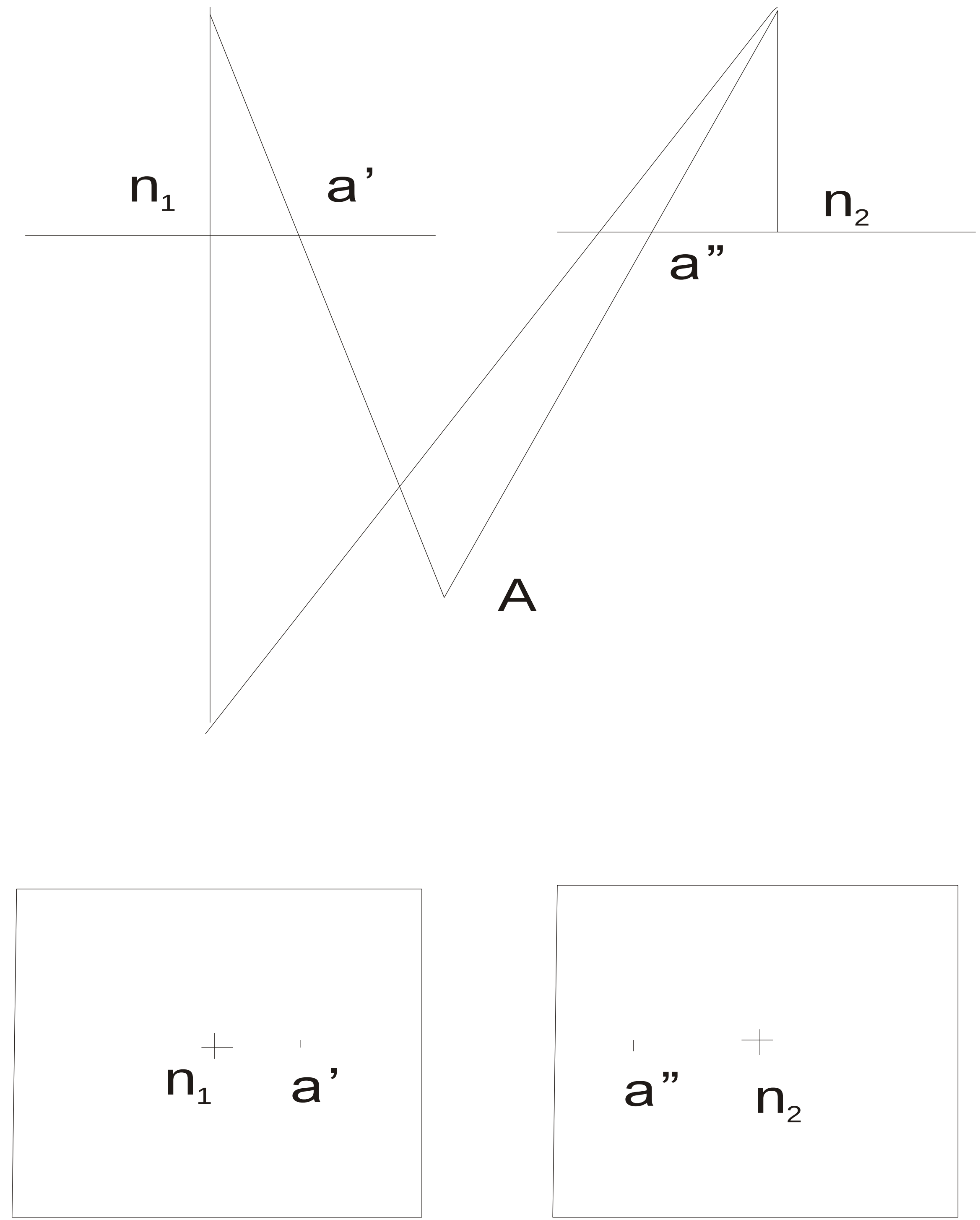
If the two photographs are placed on top of one another, the distance between the two points is the parallax.



PARALLAX

since measurements cannot be made with one photo
on top of the other, they can be separated and the parallax derived from the formula

$$P_A = n_1 n_2 - a' a''$$



Differences in parallax can be determined by measuring distances $n_1 n_2$ and a'a''.

The most accurate method is to use a **parallax bar** which simulates the application of the measuring mark. Accuracies of measurement of parallax can be of the order of
0.01mm to 0.02mm

Geometry of a Stereo Pair of Photographs

The determination of elevations from two overlapping photographs by a simple formula

Assumptions are made

(i) The photographs are taken with their optical axes exactly vertical

(ii) Aircraft stations are at the same elevation

(iii) Elevation of left hand nadir or reference point, denoted by C

(iv) Flying height over points C and A are H_C and H_A

(v) Height of points C and A above the datum are h_C and h_A

Assume

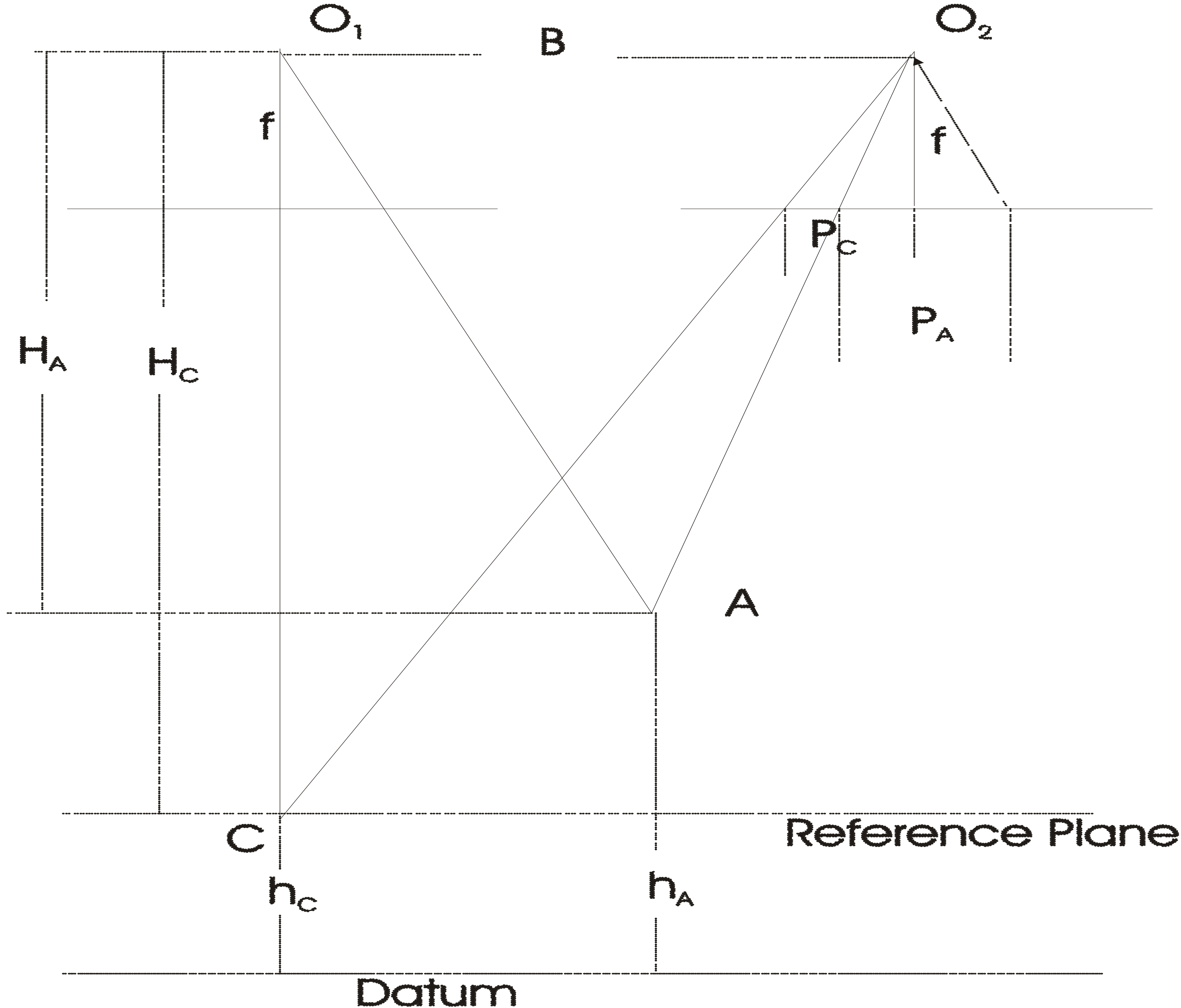
$\Delta P = P_A - P_C$

$\Delta h = h_A - h_C$

$\Delta H = H_A - H_C$

Therefore

$$\Delta h = \frac{H_C . \Delta p}{p_C + \Delta p}$$



Use of 'parallax' formula

- Subject to distortions due to tilts and variation s of flying heights of photos
- Distortions of the orders of mm across photos
- Formula demonstrates a basic characteristics of photos – relationship between heights and parallax
- Parallax is parallel to the x-direction on the photos
- Formula can be used for determining relative heights of points close together on a photo because distortions would not vary greatly